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TRANSACTION COSTS, NONFUNDAMENTAL UNCERTAINTY, AND THE EXCHANGE RATE DISCONNECT

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Exchange rates display excessive volatility and are disconnected from macroeconomic fundamentals. This paper presents a two-country general equilibrium model in which nonfundamental uncertainty ("sunspots") in part drives stochastic fluctuations in the exchange rate in a class of rational expectation equilibria. In the model, a combination of financial frictions—incomplete asset markets and a proportional transaction cost associated with trading foreign-currency-denominated bonds—breaks the tight link between exchange rates and fundamentals. Moreover, the model generates a negative Backus–Smith correlation between relative consumption and the real exchange rate, because relative prices, acting as a source of shocks as opposed to shock absorbers, directly affect relative output and generate a large wealth effect on relative consumption. Using a random walk as an example of sunspot shocks, the volatility of exchange rates relative to that of output and consumption is found to be large in the presence of nontradable goods and distribution services.

Keywords: Exchange Rates, Transaction Costs, Incomplete Asset Markets, Sunspots

1. INTRODUCTION

One of the most sustained puzzles in international economics is the well-documented excessive volatility of flexible exchange rates. In their seminal paper in 1983, Meese and Rogoff concluded that, in the short to medium run, macroeconomic variables such as the money supply, interest rates, and consumption are unable to forecast exchange rates better than a random walk model. After two decades, despite advance in econometric methods and increasing availability of high-quality data, the Meese and Rogoff result remains largely intact [Cheung et al. (2005)].[1] On the other hand, theoretical economists have had difficulty developing a general equilibrium model that can generate the observed volatility

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in the exchange rate, partly because the fundamentals are relatively stable over time. This has come to be known as the “exchange rate disconnect puzzle.”

In addition, gross international trade in assets is large, and plausibly larger than can be rationalized using standard models. As a result, many have concluded that exchange rates are driven by speculative behavior and bubbles or sunspots (e.g., market sentiments and animal spirits) in the foreign exchange markets. That is, exchange rates act as a source of destabilizing shocks as opposed to shock absorbers [Obstfeld (1985)].[2] This has led to a substantial policy discussion about what can be done to reduce speculative behavior and eliminate bubbles (for example, a “Tobin tax”). Despite widespread belief in the nonfundamental uncertainty in exchange rates, it is difficult to construct model economies in which sunspots can drive the equilibrium exchange rate fluctuations.

This paper takes the view that the nominal exchange rate, as the price of a currency, is an asset price just like stock or security prices, and hence can be influenced by nonfundamental factors. We show that it is possible to construct equilibria where exchange rate fluctuations are at least in part driven by a nonfundamental stochastic process (or “sunspot”). The equilibrium exchange rate thereby appears to have “a life of its own,” as elegantly put by Flood and Rose (1995). The two key features in our model are incomplete international asset markets and a transaction cost associated with trading foreign-currency-denominated assets.

The incomplete-asset-market assumption is widely adopted in the exchange rate literature. If international financial markets are complete, all the risk-sharing possibilities are exploited and the first-best allocation is achieved. Exchange rates are then tied to the ratio of the marginal utilities of consumption in the two countries, eliminating the possibility of exchange rate indeterminacy. As a result, equilibrium exchange rate behavior cannot be as volatile as in the data, because consumption is relatively stable.

The second feature of the model, the transaction cost, requires more explanation. Suppose there are two types of perfectly substitutable one-period non-state-contingent bonds, one denominated in the home currency and the other in the foreign currency. Households face a proportional transaction cost when they choose to hold the bond denominated in the other country’s currency. Naturally, the foreign bond is exposed to the exchange rate risk. As a result, home households choose to save in the foreign currency only if its (stochastically discounted) expected payoff weakly dominates that of the domestic one, and the transaction cost determines the spread between these two types of bonds. The same is true for foreign households. As long as the transaction cost is positive, there exists room for speculation and self-fulfilling equilibria arise. In this case, a sunspot can be injected into the exchange rate process. We show that in a framework where preferences are separable over consumption, leisure, and real money balance, a sunspot—defined as the difference between the realized exchange rate and the fundamental-determined exchange rate—can exist in the exchange rate. The only restriction the equilibrium imposes on the sunspot is that it has to follow two submartingale processes. Without the transaction cost, however, we show that the submartingale conditions imply a nominal exchange rate that is completely determined by relative monetary fundamentals, as in Devereux and Engel (2002).

We show that a random walk process in fact satisfies the two submartingale conditions, and the equilibrium exchange rate can be partly driven by a random walk.[3] In addition, the (absolute) volatility of the sunspot shocks increases with the size of the transaction cost. Although it is contrary to Tobin's speculation on the role of transaction costs in curbing the fluctuations in the financial market, this prediction of the model is consistent with the empirical evidence presented in Aliber et al. (2003).

Exchange rates are puzzling for economists not only because they are volatile, but also because they seem to have little effect on real macroeconomic variables. We show, with reasonable assumptions of nontradable goods and distribution cost, that exchange rate volatility in our model indeed is out of proportion to that of output and consumption differentials. The intensive nontradable input component in the distribution cost lowers the implied elasticity of substitution between home tradables and imported goods and significantly reduces the impact of terms of trade changes—which in our approach are mainly driven by sunspot shocks—on agents' optimal decisions. It is also well documented that the real exchange rate is strongly correlated with the nominal rate and exhibits equally large volatility. In this paper, we view the high volatility in the real exchange rate as a result of large fluctuations in the nominal exchange rate (partly driven by extrinsic uncertainty) and the deviation from PPP that is obtained with the presence of distribution costs and nontradable goods.[4] Allowing a small transaction cost, we can obtain a highly volatile nominal exchange rate and thus real exchange rate. The model also predicts that relative consumption is negatively correlated with the real exchange rate, in line with the Backus–Smith evidence on lack of risk sharing [Backus and Smith (1993)]. The relative output can also be negatively correlated with the terms of trade if the elasticity of substitution between domestic and foreign tradable goods is sufficiently low or if the nontradable distribution services are sufficiently important.

In contrast with the existing fundamentals-based models, in which productivity shocks are often the main driving force of international business cycles, our model with nonfundamental shocks can easily generate the negative Backus–Smith correlation. The reason is as follows. In most standard models, following a productivity boom, the home terms of trade and the real exchange rate depreciate; thus, the real exchange rate and relative consumption are positively correlated. The depreciation of relative prices propagates the benefits of country-specific gains abroad, improving risk sharing independent of trade in assets. In contrast, the terms of trade (as well as the real exchange rate) in this paper, driven by the sunspots, act as the source of economic fluctuations, as opposed to shock absorbers that mitigate the differences in cross-country business cycles, as in the standard model. A positive sunspot shock—which stems from "animal sprits" or changes in expectations in the international asset market—depreciates home terms of trade and the real exchange rate, lowering the relative value of home output. This produces a direct negative wealth effect that leads to lower domestic consumption relative to foreign consumption.

Starting from Keynes, many economists believe that extrinsic uncertainty plays a significant role in driving the volatility of asset prices and affecting the allocation of resources. There is a long list of theoretical work that establishes the existence of extrinsic uncertainty in closed-economy general equilibrium [e.g., Azariadis (1981); Cass and Shell (1983); Tirole (1985); Santos and Woodford (1997); Kocherlakota (2008)]. More closely related to this paper is the earlier literature on exchange rate indeterminacy. Kareken and Wallace (1981) argue that because a fiat money has no fundamental value, the competitive equilibrium exchange rate should be indeterminate as long as there are no special restrictions on the usage of different currencies. However, in their framework, once the initial exchange rate is determined, it remains constant. King et al. (1992) made a further theoretical contribution by showing that there can be equilibria where extrinsic uncertainty causes the exchange rate to fluctuate when different currencies are not perfectly substitutable, in the sense that some agents must use particular currencies as payment. In our model, currencies are potentially completely substitutable, except that transaction costs create asymmetric returns on the same assets for home and foreign investors. Manuelli and Peck (1990) consider an overlapping generations model with stochastic endowments and show that there is a multiplicity of equilibrium in which equilibrium exchange rates can take the form of many different processes. There are equilibria where a fluctuating exchange rate yields the exact same consumption as a corresponding constant exchange rate; thus, consumption and the exchange rate appear to be disconnected. Differently from the previous literature, this paper demonstrates the existence of nonfundamental components in exchange rates in a fully articulated equilibrium open-economy macro model. Some papers have also introduced noise traders in the spirit of Jeanne and Rose (2002) to achieve stochastic deviation from uncovered interest rate parity [e.g., Devereux and Engel (2002)]. Although we also explore nonfundamental exchange rate volatility, we do not assume the existence of noise traders or impose irrationalities.

Previous studies have also provided various ways to reconcile the Backus–Smith puzzle with standard international real business cycle models. One is to introduce shocks to tastes as in Stockman and Tesar (1995) or shocks to investment-specific technology as in Raffo (2009). Potentially capable of resurrecting this negative correlation between the real exchange rate and relative consumption in the presence of only productivity shocks is the model by Corsetti et al. (2008). They show that a strong wealth effect—which requires either a highly persistent productivity shock or a very low elasticity of substitution between home and foreign goods—increases the demand for domestic goods and hence the relative price. This paper takes a entirely different approach and argues that nonfundamental shocks originating from the foreign exchange market contribute to the high volatility in exchange rates and manifest themselves as exogenous terms-of-trade shocks, directly affecting relative consumption in a way that is consistent with the Backus–Smith evidence.

One strength of the current framework is that it does not need to assume low trade elasticity, which can be at odds with micro estimates and the latest macro evidence [e.g., Feenstra et al. (2012)] or persistent shocks.

The remainder of the paper is organized as follows. In Section 2, the model environment is set up. Section 3 shows that there can be equilibria in which exchange rates display randomness unrelated to fundamentals. Section 4 concludes.

2. OPEN ECONOMY WITH MONETARY SHOCKS AND SUNSPOT SHOCKS

The world economy consists of a home country and a foreign country. In each country, there are (i) an infinitely lived representative household, (ii) a tradable sector that produces nondurable consumption goods, (iii) a nontradable goods sector, (iv) a distribution sector that delivers imports and domestically produced tradable goods to households with domestic nontradable goods, and (v) a monetary authority. Variables in the foreign country are denoted by an asterisk. Goods produced in the home country are indicated by a subscript H and those originated from the foreign country by F. The following sections focus mainly on the home country, with the understanding that analogous expressions apply to the foreign country. There are an infinite number of time periods in the economy. In each time period t, the economy experiences one of finitely many exogenous states. These states specify money supply shocks and sunspot shocks that are orthogonal to each other. To keep the analysis simple and tractable, we abstract from important issues in the literature, such as sticky prices and monopolistic competition.

2.1. Households

For simplicity, assume that preference is separable in consumption, real money balances, and labor supply. Preference of a representative home household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_{t,} \frac{M_t}{P_t}, L_t\right),$$

where

$$U\left(C_{t,} \frac{M_t}{P_t}, L_t\right) = \left[\frac{C_t^{1-\omega}}{1-\omega} + \chi \ln\left(\frac{M_t}{P_t}\right) - \frac{\phi}{1+\psi} L_t^{1+\psi}\right], \tag{1}$$

where $\beta < 1$, C_t is home aggregate consumption, L_t is labor supply, and M_t/P_t is the home real money balance. Preferences over consumption, real money balance, and labor supply are separable. The household consumes a basket of nontradable goods C_{N} and tradable goods C_{T} which consist of domestically produced tradables C_{H} and imports from the foreign country C_{F}. The elasticity of substitution between tradable and nontradable goods is denoted by ρ, and that between home- and foreign-produced tradable goods by θ. The domestic nontradable goods are used for both consumption and distribution services. The consumption indices for aggregate goods and tradable goods are

$$C_t = \left[\alpha_{\mathrm{T}}^{\frac{1}{\rho}} C_{\mathrm{T},t}^{\frac{\rho-1}{\rho}} + (1-\alpha_{\mathrm{T}})^{\frac{1}{\rho}} C_{\mathrm{N},t}^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \tag{2}$$

$$C_{\mathrm{T},t} = \left[\alpha_{\mathrm{H}}^{\frac{1}{\theta}} C_{\mathrm{H},t}^{\frac{\theta-1}{\theta}} + (1-\alpha_{\mathrm{H}})^{\frac{1}{\theta}} C_{\mathrm{F},t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

where $\alpha_{\mathrm{H}} > 1/2$ implies home bias. The associated price index for the aggregate consumption P_t and that for the tradable consumption goods $P_{\mathrm{T},t}$ are

$$P_t = \left[\alpha_{\mathrm{T}} P_{\mathrm{T},t}^{1-\rho} + (1-\alpha_{\mathrm{T}}) P_{\mathrm{N},t}^{1-\rho}\right]^{\frac{1}{1-\rho}}, \tag{4}$$

$$P_{\mathrm{T},t} = \left[\alpha_{\mathrm{H}} P_{\mathrm{H},t}^{1-\theta} + (1-\alpha_{\mathrm{H}}) P_{\mathrm{F},t}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{5}$$

where $P_{\mathrm{N},t}$ is the price of home nontradable goods, $P_{\mathrm{H},t}$ is the price of home tradable goods, and $P_{\mathrm{F},t}$ is the price of imports in the home currency.

The international financial market is incomplete, and home and foreign households are allowed to trade two nominal risk-free bonds $B_{\mathrm{H},t}$ and $B_{\mathrm{F},t}$, denominated in domestic and foreign currencies, respectively. But they can only borrow in bonds denominated in domestic currency. In other words, $B_{\mathrm{F},t} \geq 0$, $B_{\mathrm{H},t}^{*} \geq 0$. The bond prices are denoted by q_t in the home currency and q_t^{*} in the foreign currency. An important feature of this model is that households face a transaction cost when undertaking a position in the foreign exchange market. Thus, a borrower pays a fixed premium on the foreign interest, whereas a lender suffers from having less foreign interest income. This cost is equivalent to a service or handling fee charged on a foreign currency account or a bid–ask spread in the foreign currency.[5]

The sequence of events within a given time period is as follows. First, the sunspot and monetary shocks are realized and observed by all agents in the economy. Households enter the period with domestic cash balance $M_{\mathrm{H},t}$, domestic nominal bond holding $B_{\mathrm{H},t}$, and foreign bond holding $B_{\mathrm{F},t}$. The value of the foreign bond holding is reevaluated at the current nominal exchange rate S_t. The households also receive a cash transfer $\mathrm{TR}_t = M_t - M_{t-1}$ from the home monetary authority. Then the households decide on their consumption, money holding, labor supply, and bond holding in the financial markets. They have to pay a transaction cost of γ (> 0) percent of the foreign bond trading. The aggregate transaction cost, $\mathrm{TB}_t = B_{\mathrm{F},t+1} q_t^{*} S_t \gamma \geq 0$, is then rebated to domestic households in a lump-sum fashion. In the production sectors, firms produce respective goods and earn profits Π_t, which are redistributed to the households. The home household's flow budget constraint is given by

$$\begin{aligned} &P_t C_t + M_{\mathrm{H},t+1} + B_{\mathrm{H},t+1} q_t + B_{\mathrm{F},t+1} q_t^{*} S_t (1+\gamma) \\ &\quad \leq M_{\mathrm{H},t} + B_{\mathrm{H},t} + B_{\mathrm{F},t} S_t + \Pi_t + W_t L_t + \mathrm{TR}_t + \mathrm{TB}_t. \end{aligned} \tag{6}$$

2.2. Goods Producers and Distribution Service

In each country, there are two production sectors (one for tradables and the other for nontradables) and a distribution sector. Firms are perfectly competitive, and production of consumption goods is carried out using linear technologies $Y_t = A_t L_t$, where A_t denotes productivity. Prices are flexible and set in the consumers' currency (i.e., local currency pricing is used). After the state is realized in period t, tradable-goods producers choose a wholesale price $P^p_{\mathrm{H},t}$ for the domestic market and $P^{p*}_{\mathrm{H},t}$ for the foreign market in order to maximize profit. Because there is no impediment to trade, the law of one price holds at the producer level each period, and perfect competition implies that prices are determined by the marginal cost:

$$P^p_{\mathrm{H},t} = S_t P^{p*}_{\mathrm{H},t} = \frac{W_t}{A^{\mathrm{T}}_t}. \tag{7}$$

Similarly, the price of nontradable goods is given by $P_{\mathrm{N},t} = W_t / A^{\mathrm{N}}_t$.

Burstein et al. (2003) emphasize the role of local distribution services, such as wholesale and retail services, advertisement costs, market penetration, and local transportation in delivering consumption goods to households. Similarly, we assume that the distribution sector buys tradable goods from producers and charges η units of local nontradable goods to deliver one unit of tradable goods to households. Free entry implies zero profit, and the distribution cost creates a wedge between the producer and consumer prices:

$$P_{\mathrm{H},t} = P^p_{\mathrm{H},t} + \eta P_{\mathrm{N},t}, \quad P_{\mathrm{F},t} = P^p_{\mathrm{F},t} + \eta P_{\mathrm{N},t}. \tag{8}$$

2.3. Money Supply

Government changes the money supply through direct transfers. For simplicity, we assume that money supply follows a simple random walk stochastic process:

$$M_{t+1} = g_t M_t \text{ and } \ln g_t = \varepsilon_{g,t} \sim N(0, \sigma^2_g). \tag{9}$$

This implies that

$$E_t \frac{M_t}{M_{t+1}} \equiv \frac{1}{g} = \exp\left(\frac{\sigma^2_g}{2}\right) \text{ for all } t. \tag{10}$$

The stochastic process of money supply in the foreign country is analogous. Home and foreign money supplies are uncorrelated.

3. CHARACTERIZING COMPETITIVE EQUILIBRIUM

We define equilibrium as follows:

DEFINITION 1. *Given the initial holdings of bonds and money* ($B_{\mathrm{H},0}$, $B_{\mathrm{F},0}$, M_0, M_0^*)*, an equilibrium is a set of prices* $\{S_t, P_t, P_t^*, W_t, W_t^*, q_t, q_t^*\}_{t=0}^{\infty}$*, consumption allocations* $\{C_{\mathrm{H},t}, C_{\mathrm{F},t}, C_{\mathrm{H},t}^*, C_{\mathrm{F},t}^*, C_{\mathrm{N},t}, C_{\mathrm{N},t}^*\}_{t=0}^{\infty}$*, portfolio choices* $\{B_{\mathrm{H},t}, B_{\mathrm{H},t}^*, B_{\mathrm{F},t}, B_{\mathrm{F},t}^*\}_{t=0}^{\infty}$*, labor supply* $\{L_t, L_t^*\}_{t=0}^{\infty}$*, and currency holdings* $\{M_{\mathrm{H},t}, M_{\mathrm{F},t}^*\}_{t=0}^{\infty}$ *such that*

(i) *Given prices and history* s^t*, households maximize the lifetime utility subject to (a) the budget constraints, (b) nonnegative money holding conditions* $M_{\mathrm{H},t} \geq 0$, $M_{\mathrm{F},t}^* \geq 0$*, (c) foreign currency borrowing constraints* $B_{\mathrm{F},t} \geq 0$, $B_{\mathrm{H},t}^* \geq 0$*, and (d) the transversality condition* $\lim_{T\to\infty} E_t \beta^{t+T} \frac{U_{C,t+T}}{P_{t+T}} (M_{t+T} + B_{\mathrm{H},t+T} + B_{\mathrm{F},t+T}^* S_{t+T}) = 0$.

(ii) *The goods market clears for traded goods and nontraded goods:*

$$C_{\mathrm{H},t} + C_{\mathrm{H},t}^* = A_t^{\mathrm{T}} L_t^{\mathrm{T}}, \quad (C_{\mathrm{H},t} + C_{\mathrm{F},t})\eta + C_{\mathrm{N},t} = A_t^{\mathrm{N}} L_t^{\mathrm{N}}, \tag{11}$$

$$C_{\mathrm{F]},\mathrm{t}} + C_{\mathrm{F},t}^* = A_t^{*\mathrm{T}} L_t^{*\mathrm{T}}, \quad (C_{\mathrm{H},t}^* + C_{\mathrm{F},t}^*)\eta + C_{\mathrm{N},t}^* = A_t^{*\mathrm{N}} L_t^{*\mathrm{N}}. \tag{12}$$

(iii) *The bond market clears:* $B_{\mathrm{H},t} + B_{\mathrm{H},t}^* = 0$, $B_{\mathrm{F},t}^* + B_{\mathrm{F},t} = 0$.

3.1. Exchange Rates and Nonfundamental Uncertainty

Because the representative agent's problem involves maximizing a concave function over a nonempty, compact constraint set, the first-order conditions are necessary and sufficient. Solving the household's problem, the optimality conditions with respect to labor supply and real money balance are

$$\frac{W_t}{P_t} = \phi L_t^{\psi} C_t^{\omega}, \tag{13}$$

$$\frac{M_t}{P_t} = \frac{C_t^{\omega}}{1 - q_t}. \tag{14}$$

The first equation implies that the real wage should equal the marginal rate of substitution between consumption and leisure. The second equation captures the trade-off between money holding and consumption and can be equivalently written as $U_{M,t} R_t = (R_t - 1)\frac{U_{c,t}}{P_t}$, where $R_t = 1/q_t$ is the gross nominal interest rate. Therefore, the marginal benefit of saving one additional unit of the domestic currency equals the marginal cost of forgoing the interest on this one unit of currency. Substituting the equilibrium bond price, $q_t = E_t(Q_{t,t+1})$, where $Q_{t,t+1} = \beta \frac{C_{t+1}^{-\omega}/P_{t+1}}{C_t^{-\omega}/P_t}$, into the first-order condition of money balance, iterating forward, and imposing the transversality conditions, we get the marginal utility of real consumption in terms of the stream of optimal money demand as

$$\frac{C_t^{-\omega}}{P_t} = \frac{1}{M_t} + \beta E_t \frac{C_{t+1}^{-\omega}}{P_{t+1}} = E_t \Sigma_{i=0}^{\infty} \beta^i \frac{1}{M_{t+i}}. \tag{15}$$

Random walk monetary shocks, together with real money balance in logarithms in the utility function, implies a time-invariant interest rate $R_t = g/\beta$. This implies that the state-contingent value of domestic currency delivered in period $t + 1$ (i.e., the pricing kernel) is determined by the stochastic money growth rate, $Q_{t,t+1} = \beta \frac{M_t}{M_{t+1}} \frac{1-q_t}{1-q_{t+1}} = \frac{\beta}{g_{t+1}}$. For the home (foreign) household, the bond denominated in foreign (home) currency is a risky asset, as its value fluctuates with the nominal exchange rate. The discounted expected excess return of the risky asset equals zero in the equilibrium with nonzero bond holdings. Therefore,

$$E_t Q_{t,t+1} \left[R_t^* \frac{S_{t+1}}{S_t(1+\gamma)} - R_t \right] = 0, \tag{16}$$

and from the foreign households' point of view,

$$E_t Q_{t,t+1}^* \left[R_t \frac{S_{t+1}^{-1}}{S_t(1+\gamma)} - R_t^* \right] = 0. \tag{17}$$

Intuitively, currency is an asset because it is storable and can be invested. The price of an asset (i.e., the price of the foreign currency in terms of the domestic currency, S_t, and the price of the domestic currency in terms of the foreign currency, S_t^{-1}) is no determined simply by current flow supply and demand. Instead, it is shown in the preceding to follow a quasi-martingale process, which means that the price of the asset is the expected discounted value of its future payoff. For the home household, the payoff of holding the bond denominated in foreign currency is governed by the growth rate of the nominal exchange rate and foreign interest rate, whereas for the foreign household, the payoff to the bond denominated in home currency is governed by the growth rate of the reciprocals of the nominal exchange rate, both adjusted by the transaction cost. Given the equilibrium marginal rate of substitution and time-invariant interest rates, equations (16) and (17) yield

$$E_t \frac{S_{t+1}}{g_{t+1}} = \frac{S_t}{g^*}(1+\gamma), \quad E_t \frac{S_{t+1}^{-1}}{g_{t+1}^*} = \frac{S_t^{-1}}{g}(1+\gamma). \tag{18}$$

PROPOSITION 2. *With an incomplete asset market and a transaction cost* γ (> 0), *there exists a class of equilibria in which the nominal exchange rate is driven by two components, fundamentals* (M_t/M_t^*) *and a sunspot* (ν_t). *Specifically,* $S_t = \frac{M_t}{M_t^*}\nu_t$, *where* ν_t *is a sunspot that is unrelated to preference, money supply, and productivity. The sunspot component follows a forward-looking stochastic process and must satisfy the following restrictions:* $E_t\nu_{t+1} = \nu_t(1+\gamma)$ *and* $E_t\nu_{t+1}^{-1} = \nu_t^{-1}(1+\gamma)$.

The strategy to prove this proposition is "guess and verify" ; i.e., a candidate equilibrium is proposed and shown to satisfy the equilibrium conditions. Let $\nu_t = S_t/S_t^f$ denote the deviation of the equilibrium nominal exchange rate from its fundamental value $S_t^f = M_t/M_t^*$.[6] Plug $S_t = \frac{M_t}{M_t^*}\nu_t$ back into (18). The equilibrium intertemporal conditions become $E_t(\frac{M_t}{M_{t+1}^*}\nu_{t+1}) = \frac{M_t}{M_t^* g^*}\nu_t(1+\gamma)$ and $E_t(\frac{M_t^*}{M_{t+1}}\nu_{t+1}^{-1}) = \frac{M_t^*}{M_t g^*}\nu_t^{-1}(1+\gamma)$. Given the random-walk nature of the money supply, these equations are reduced to

$$E_t\nu_{t+1} = \nu_t(1+\gamma) \text{ and } E_t\nu_{t+1}^{-1} = \nu_t^{-1}(1+\gamma). \qquad \textbf{(19)}$$

Therefore, in this specific class of equilibria, the sunspot is necessarily constrained according to two submartingale conditions implied by the optimal intertemporal substitution, even though it can take arbitrary functional forms. Intuitively, because the rate of return on the foreign asset is risky and is also associated with a positive transaction fee, for this equilibrium to be sustained investors must see the opportunity for speculation and a self-fulfilling equilibrium (sunspots) must arise. Because of the wedge created by the proportional transaction cost, home (foreign) investors expect (rationally) that the bonds denominated in foreign (home) currency will pay off better ($E_t\frac{\nu_{t+1}}{\nu_t} > 1$ and $E_t\frac{\nu_{t+1}^{-1}}{\nu_t^{-1}} > 1$), and both optimally choose nonzero bond holdings in the currency of the other country.

To see the role of the positive transaction cost mechanically, from (19) it follows that $\text{Cov}_t(\nu_{t+1}, \nu_{t+1}^{-1}) = 1 - (1+\gamma)^2 < 0$ for $\gamma > 0$. Otherwise, if $\gamma = 0$, by Jensen's inequality, $\text{Cov}_t(\nu_{t+1}, \nu_{t+1}^{-1}) = 0$, implying that $\{\nu_t\}_{t=0}^{\infty}$ has to be deterministic and the nominal exchange rate is proportional to the relative exogenous money supply. That is, $S_t = M_t/M_t^*$.

In our specific framework, $\sigma^2(\ln S_t) = \sigma^2(\ln M_t - \ln M_t^*) + \sigma^2(\ln \nu_t)$, the fluctuations in the nominal exchange rate are *in part* caused by the extrinsic uncertainty regardless of how prices are set or how production is carried out. The nominal exchange rate is a financial variable and related only to the monetary variables and foreign exchange market elements (such as market sentiment or "animal spirits").

Our analysis so far has highlighted how the nominal exchange rate can (potentially) be jointly determined by monetary fundamentals and sunspots. But note that the proof is not complete until we specify the rest of the economy. Especially, we need to show that goods market trading would not eliminate nonfundamental uncertainties in the economy. A related question is, Can sunspots in the nominal exchange rate be an important source of shocks that drive large swings in output and consumption? This question will be answered in Sections 3.2 and 3.3.

On "Siegel's paradox." To further illustrate the role of the financial transaction cost in generating nonfundamental randomness in the nominal exchange rate, consider "Siegel's Paradox." Siegel (1972) shows that if the forward dollar price of the Euro F_t perfectly reflects the expected future spot rate S_{t+1}, so that $F_t = E_tS_{t+1}$, then by Jensen's inequality the forward Euro price of the dollar F_t^{-1} would be $F_t^{-1} < E_tS_{t+1}^{-1}$. Hence, if investors are risk-neutral, then there exists no forward exchange rate that rules out expected profit opportunities from speculating on future exchange rates. Engel (1984, 1999), however, emphasizes that investors are interested in real profits rather than nominal ones. If the states of the world in which one makes a profit in domestic currency are precisely the ones where domestic prices of goods increase, there is no real profit in the forward market.

This model is consistent with this insight. In equilibrium, the forward rate is $F_t = S_t R_t / R_t^*$. Consider first $\gamma = 0$. Equations (16) and (17) imply that

$$F_t = \frac{E_t Q_{t,t+1} S_{t+1}}{E_t Q_{t,t+1}} = E_t \beta \frac{C_{t+1}^{-\omega}}{C_t^{-\omega}} \frac{S_{t+1}}{P_{t+1}} / E_t \beta \frac{C_{t+1}^{-\omega}}{C_t^{-\omega}} \frac{1}{P_{t+1}}, \tag{20}$$

where $\beta \frac{C_{t+1}^{-\omega}}{C_t^{-\omega}}$ is the stochastic discount factor for *real* values and $1/P_{t+1}$ is the purchasing power of the domestic currency in the next period. In our model, if investors are risk-neutral (i.e., $\omega = 0$), $\text{cov}_t(P_{t+1}^{-1}, S_{t+1}) < 0$, because under flexible pricing, increasing money supply depreciates the domestic currency and at the same time raises prices. It follows that $F_t = E_t S_{t+1} + \text{cov}_t(P_{t+1}^{-1}, S_{t+1}) < E_t S_{t+1}$. Similarly, $F_t^{-1} < E_t S_{t+1}^{-1}$. Therefore, both domestic and foreign investors can expect to make nominal profits in their own currencies, but no real profit measured in their own baskets of goods.

In our model, we can make a stronger statement with a small positive transaction cost. That is, we can still solve the paradox even when $\text{cov}_t(Q_{t,t+1}, S_{t+1}) = \text{cov}_t(Q_{t,t+1}^*, S_{t+1}^{-1}) = 0$. Consider an extreme case in which there is no fundamental uncertainty; i.e., $g_t = g$ and $g_t^* = g^*$ $\forall t$. In this case, the forward rate satifies $E_t^{-1} S_{t+1}^{-1}(1+\gamma) = F_t = E_t S_{t+1}/(1+\gamma)$ and S_t is driven only by the stochastic sunspots. There are no real (or nominal) profits in the forward market, simply because the expected returns of the same asset for domestic and foreign investors are asymmetric because of the transaction costs. The transaction costs create a wedge that allows the nominal exchange rate to be stochastic even when fundamentals are deterministic.

Random walk sunspots and the "Tobin Tax." We have shown that the exchange rate can incorporate an exogenous, nonfundamental process, ν_t, with the two submartingale conditions as restrictions. Here, we investigate the relationship between the transaction cost and the volatility of the sunspot process by considering a special candidate equilibrium process for the sunspots.

Meese and Rogoff (1983) show that, among industrial countries, floating exchange rates behave like random walk processes. That is, today's spot rate is the best guess of tomorrow's: $\ln S_{t+1} = \ln S_t + \zeta_t$, where ζ_t is an i.i.d. normal random variable. An example of the sunspot process $\{\nu_t\}_{t=0}^{\infty}$ that generates a random walk nominal exchange rate can be given by

$$\ln \nu_{t+1} = \ln \nu_t + \zeta_t \text{ and } \zeta_t \sim N(0, \sigma_\nu^2). \tag{21}$$

Equilibrium conditions (19) thus imply that $\sigma_\nu^2 = 2\ln(1+\gamma)$. Given that $S_t = \frac{M_t}{M_t^*}\nu$, the growth rate of the nominal exchange rate is now lognormally distributed, and $E_t \frac{S_{t+1}}{S_t} = \exp(\frac{\sigma_S^2}{2}) = (1+\gamma)\frac{g^*}{g}$. Equivalently,

$$\sigma^2(S_t) = 2\ln\left[(1+\gamma)\frac{g^*}{g}\right]. \tag{22}$$

Therefore, the volatility of the exchange rate is an increasing function of the transaction cost.[7]

This positive relationship between the volatility of the nominal exchange rate and the size of the transaction cost is related to an earlier debate over the role of the "Tobin tax." Tobin (1978) suggests imposing a tax on all foreign exchange transactions to decrease the volatility of prices. The argument is that a speculator who traded more frequently would be hurt more by this transaction tax. Empirically, Aliber et al. (2003) present evidence showing that volatility increases with the level of transaction costs faced by marginal investors, contrary to Tobin's speculation. Our model prediction is in line with this empirical observation. A higher transaction cost leaves more room for speculation and allows larger fluctuations of sunspots.

3.2. Exchange Rates and Consumption Differentials

For the sake of clarity, we first consider a simple scenario when labor supply is perfectly elastic, i.e., $\psi = 0$ [equivalent to the indivisible labor as in Hansen (1985)]. In addition, to keep the analysis focused, we assume productivity levels are the same across tradable and nontradable sectors.

Under the simplifying assumptions, the producer's price of the home-produced product is equivalent to the price of the nontradable and both are given by the unit labor cost under the zero profit condition: $P^p_{\mathrm{H},t} = P_{\mathrm{N},t} = W_t/A_t$, where the equilibrium nominal wage $W_t = \phi L_t^{\psi} C_t^{\omega} P_t = \frac{\phi}{\chi} L_t^{\psi} M_t(1-q_t)$. The fluctuations in the terms of trade at producer prices are thereby completely driven by the sunspot component (ν_t) of the nominal exchange rate. That is,

$$\tau_t = \frac{P^p_{\mathrm{F},t}}{P^p_{\mathrm{H},t}} = \frac{S_t W_t^*}{W_t}\frac{A_t}{A_t^*} = \frac{(1-q_t^*)A_t M_t^*}{(1-q_t)A_t^* M_t} S_t = \frac{1-\beta/g^*}{1-\beta/g}\frac{A_t}{A_t^*}\nu_t. \quad \mathbf{(23)}$$

Without sunspots, the equilibrium terms-of-trade movements exactly offset the changes in relative productivity (if there are any) and help to achieve risk sharing irrespective of trade in financial markets, which is similar to the point emphasized by Cole and Obstfelt (1991) and Corsetti and Pesenti (2001, 2005). With a perfectly elastic labor supply, the nominal labor cost is proportional to the money supply, offsetting the direct effects of monetary shocks but not the sunspot shocks. Therefore, in the absence of productivity shocks, sunspots manifest themselves as exogenous terms-of-trade shocks.

Putting the equilibrium conditions (7), (13), and (23) together, we can link the relative consumption directly to the terms of trade and the real exchange rate as follows:

$$\left(\frac{C_t}{C_t^*}\right)^{\omega} = \frac{W_t/P_t}{W_t^*/P_t^*} = \frac{P^p_{\mathrm{H},t}A_t/P_t}{P^{p*}_{\mathrm{F},t}A_t^*/P_t^*} = \frac{A_t}{\tau_t A_t^*}\frac{S_t P_t^*}{P_t} = \frac{A_t}{\tau_t A_t^*}\mathrm{RER}_t. \quad \mathbf{(24)}$$

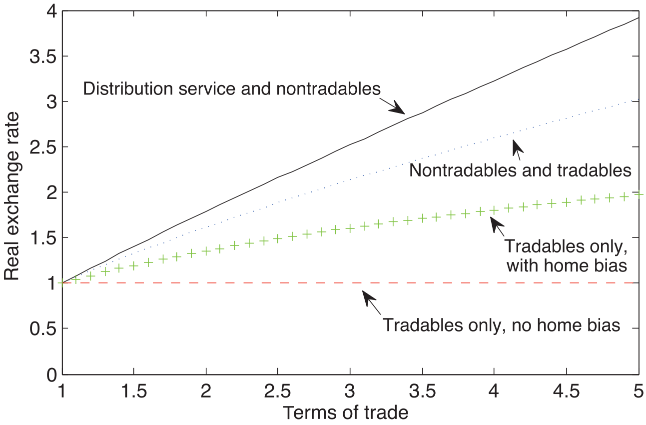

FIGURE 1. Constructed real exchange rate.

Substituting (23) into (24), we get

$$\mathrm{RER}_t = \left(\frac{C_t}{C_t^*}\right)^{\omega} \frac{1-\beta/g^*}{1-\beta/g} \nu_t. \qquad \mathbf{(25)}$$

Equation (25) implies that under simplifying assumptions, the real exchange rate in this economy not only is linked to the relative consumption, but also is driven by nonfundamental uncertainties. Now whether the real exchange rate displays volatility in excess of the relative consumptions depends on the effect of the sunspot shocks on the consumption differential.

Before investigating the impact of sunspots on consumption, we first study the response of the real exchange rate. The salient feature of the real exchange rate in the data is that it is strongly correlated with the nominal exchange rate and almost equally volatile. One simple and realistic way to achieve this is to introduce nontradable goods and distribution costs [Burstein et al. (2003)], so that the relative consumer price index can hardly offset the nominal exchange rate fluctuations.[8]

With the distribution service and nontradable goods, the real exchange rate is a highly nonlinear function of the terms of trade.[9] Figure 1 shows that given a reasonable set of parameter values ($\alpha_{\mathrm{H}} = 0.72, \theta = 1.5, \rho = 0.74$), the inclusion of the distribution services and nontradable sectors substantially improves the response of the real exchange rate with respect to the sunspot (i.e., the line gets closer to 45°). The intuition is as follows. In an extreme case where every product is nontradable, the real exchange rate fluctuation is completely driven by the terms of trade, which is the relative unit labor cost measured in the same currency. Adding distribution services increases the importance of the nontradable component in the tradable goods prices, bringing the real exchange rate closer to the terms of trade and the sunspot.

Because this paper focuses on nonfundamental uncertainties, we assume productivity is constant. By taking a log-linear approximation of the equation relating the real exchange rate to the terms of trade/sunspot around a steady state symmetric equilibrium, we get[10]

$$\widetilde{\mathrm{rer}}_t = [1 + \Omega(1 - \Theta)(2\alpha_{\mathrm{H}} - 2)]\tilde{\tau}_t = [1 + \Omega(1 - \Theta)(2\alpha_{\mathrm{H}} - 2)]\tilde{\nu}_t, \qquad \textbf{(26)}$$

where $\Omega \equiv \frac{\alpha_{\mathrm{T}}\bar{P}^{T1-\rho}}{\alpha_{\mathrm{T}}\bar{P}^{T1-\rho}+(1-\alpha_{\mathrm{T}})\bar{P}^{N1-\rho}} \in [0, 1]$ is the relative price of tradables in the aggregate price index and $\Theta \equiv \eta\frac{\bar{P}_{\mathrm{N}}}{\bar{P}_{\mathrm{H}}} \in [0, 1]$ denotes the size of the distribution margin in the deterministic steady state.[11] The real exchange rate fluctuates with the nonfundamental shocks in the same direction because $\alpha_{\mathrm{H}} > 1/2$, and the correlation between the two increases with the distribution margin and decreases with the relative importance of tradable goods price in the composite price index.

Combining the log-linearized (25) with (26), we can write the consumption differential in terms of the terms-of-trade fluctuations and real exchange rate as

$$\tilde{c}_t - \tilde{c}_t^* = \frac{(1 - \Theta)\Omega(2\alpha_{\mathrm{H}} - 2)}{\omega}\tilde{\nu}_t = \frac{(1 - \Theta)\Omega(2\alpha_{\mathrm{H}} - 2)}{\omega[1 + \Omega(1 - \Theta)(2\alpha_{\mathrm{H}} - 2)]}\widetilde{\mathrm{rer}}_t. \qquad \textbf{(27)}$$

Because $0 < \alpha_{\mathrm{H}} \leq 1$, regardless of the importance of the distribution services and nontradables we obtain a negative Backus–Smith correlation: the real exchange rate appreciation is associated with a rise in home aggregate consumption relative to foreign aggregate consumption.

Why does our model predict such a different relationship between the consumption differential and the real exchange rate from those in most of the fundamental-based (incomplete market) models? Because the sources of shocks are different. When the economic fluctuation is driven by fundamental shocks such as productivity shocks, a positive shock to home productivity leads to relatively lower home tradable prices [see equation (23)]. As shown in Corsetti et al. (2008), only when the negative domestic wealth effect is large enough (either by a low elasticity of substitution between the two tradables or by bond trading in anticipation of future output loss) so that home aggregate consumption falls in the short run can a negative Backus–Smith correlation be achieved. In contrast, relative price changes in this model are driven by nonfundamental shocks. In other words, one can interpret the relative price change itself as a source of uncertainty. When there is a positive shock to the sunspot, the relative price of home output falls and generates a large negative wealth effect, leading to a lower real wage and lower domestic consumption.

More specifically, consider a simpler case where there are only tradable goods. Combining the definition of the price index, zero profit condition, and optimal labor supply decision implies that home consumption depends on its technology and the relative price:

$$\phi C_t^{\omega} = \frac{W_t}{P_t} = \frac{P_{\mathrm{H}}}{P_t}A_t = \frac{A_t}{[\alpha_{\mathrm{H}} + (1 - \alpha_{\mathrm{H}})\tau_t^{1-\theta}]^{\frac{1}{1-\theta}}}. \qquad \textbf{(28)}$$

Similarly,

$$\phi C_t^{*\omega} = \frac{\tau_t A_t^*}{[\alpha_{\mathrm{F}} \tau_t^{1-\theta} + (1 - \alpha_{\mathrm{F}})]^{\frac{1}{1-\theta}}}. \tag{29}$$

Therefore,

$$\left(\frac{C_t}{C_t^*}\right)^{\omega} = \frac{A_t}{\tau_t A_t^*} \frac{[\alpha_{\mathrm{F}} \tau_t^{1-\theta} + (1 - \alpha_{\mathrm{F}})]^{\frac{1}{1-\theta}}}{[\alpha_{\mathrm{H}} + (1 - \alpha_{\mathrm{H}}) \tau_t^{1-\theta}]^{\frac{1}{1-\theta}}}. \tag{30}$$

In the standard models, fluctuations in τ_t are induced by changes in A_t/A_t^*, and exactly offset the impact of relative productivity movements under our simplifying assumptions. Relative consumption unambiguously increases with the terms of trade. However, when sunspot shocks exist, they drive the deviation of τ_t from A_t/A_t^*, and relative consumption decreases with the terms of trade (sunspots).

In addition, the impact on the relative consumption of the real exchange rate fluctuations—measured by the volatility of the consumption differential relative to the volatility of the real exchange rate—decreases with the size of the distribution margin and the significance of the nontradable price in the aggregate price index. Optimization within a country requires that the ratio of the marginal utilities of two consumption goods equal the relative prices: $\frac{C_{\mathrm{H},t}}{C_{\mathrm{F},t}} = \frac{\alpha_{\mathrm{H}}}{1-\alpha_{\mathrm{H}}} (\frac{P_{\mathrm{H},t}^p + \eta P_{\mathrm{N},t}}{P_{\mathrm{F},t}^p + \eta P_{\mathrm{N},t}})^{-\theta}$ or $\tilde{c}_{\mathrm{H},t} - \tilde{c}_{\mathrm{F},t} = \theta(1 - \Theta)\tilde{\tau}_t$. Thus, sunspots drive changes in the terms of trade, causing "expenditure switching" between domestic- and foreign-produced tradables. Without distribution costs or nontradables, changes in terms of trade directly affect relative consumption between Home and Foreign. In contrast, distribution costs create a wedge between producer and consumer prices, limiting the expenditure-switching role of the terms of trade. In the presence of nontradables, the households also choose between tradable and nontradable goods in response to terms-of-trade changes. If consumers substitute tradable goods for nontradable goods, the demand for nontradable goods will actually be higher as the demand for nontradable distribution services increases. Therefore, distribution margin and nontradables play an important role in mitigating the effect of real exchange rate movements on the consumption differential.

3.3. Output Differential

To derive a relationship between relative output and sunspot shocks, it is helpful first to investigate how sunspots affect net exports. Taking a linear approximation of the home household's budget constraint (after imposing money supply equilibrium, rebate of the transaction cost, and the equilibrium relationship for profits) and using the relative price differentials, we can express the percentage deviation from the steady state net exports-to-GDP ratio as

$$\delta \frac{\tilde{nx}_t}{y_t} = (b - 1)\tilde{\tau}_t - (\tilde{c}_t - \tilde{c}_t^*), \tag{31}$$

where $\tilde{nx}_t$ denotes changes in net exports, $\delta = [(1-\alpha_{\mathrm{H}})\Omega(1-\Theta)]^{-1}$, and $b = 2(1-\Theta)[\rho(1-\Omega)(1-\alpha_{\mathrm{H}}) + \alpha_{\mathrm{H}}\theta]$. Substituting equation (27) into (31) yields[12]

$$\delta \frac{\tilde{nx}}{y} = \left(b + \frac{2}{\omega\delta} - 1\right)\tilde{\tau}_t. \tag{32}$$

Meanwhile, $Y_t = (C_{\mathrm{H},t} + C^*_{\mathrm{H},t}) + \frac{P_{\mathrm{N},t}}{P^p_{\mathrm{H},t}}[(C_{\mathrm{H},t} + C_{\mathrm{F},t})\eta + C^{\mathrm{N}}_t]$. The relationship between relative output, the terms of trade, and net exports as a fraction of GDP is

$$\tilde{y}_t - \tilde{y}^*_t = \left(b + \frac{2}{\delta} - 1\right)\tilde{\tau}_t + (2-\delta)\frac{\tilde{nx}_t}{y}. \tag{33}$$

This relationship holds independent of the international asset market structure or the preference specification. Substituting (32) into (33), we have

$$\tilde{y}_t - \tilde{y}^*_t = \frac{2}{\delta}\left[b - \frac{1}{\omega}\left(1 - \frac{2}{\delta}\right)\right]\tilde{\tau}_t. \tag{34}$$

where $\delta > (1-\alpha_{\mathrm{H}})^{-1} > 2$.

Corsetti et al. (2008) show that the correlation between output and terms of trade tends to be negative or small for most OECD countries.[13] Our model suggests that sunspot shocks can drive a negative correlation between the relative output and the terms of trade if home tradable goods and foreign tradable goods are not easily substitutable or when there exists an intensive nontradable input component in the distribution cost. First, other things equal, when θ is sufficiently low (b is relatively small), the terms of trade deterioration (i.e., the increase in τ_t) decreases domestic output relative to foreign output. Home consumption of domestic products compared to home consumption of foreign products unambiguously increases because of the expenditure-switching effect. However, foreign consumption of home products relative to the consumption of foreignn product may drop despite the fact that domestic goods become cheaper, because the improvement in the foreign terms of trade increases the value of foreign output and income, inducing a positive wealth effect, which encourages them to consume their own products with the home bias. When the wealth effect dominates the substitution effect, the correlation between the output differential and the terms of trade is negative.

Second, when distribution sectors become important (i.e., Θ approaches one, b approaches zero, and δ approaches infinity), home terms-of-trade deterioration tends to decrease net exports and relative output, because the *implied* elasticity of substitution between home- and foreign-produced tradable goods becomes lower as the distribution of tradable goods involves using more nontradables. As argued before, the wealth effect dominates and a terms-of-trade deterioration induces home output to decrease relative to foreign output.

Note that (31) and (33) are derived independent of the international asset market structure. Suppose the market is complete; then by substituting the relationship

TABLE 1. Illustrative evidence of the effect of key parameters

	Benchmark			Variation on the parameters ($\psi = 1, \theta = 1.5$)			
	$\psi = 1$			Labor elasticity		No distribution nor nontradable	Transaction cost
Statistics	$\theta = 0.5$	$\theta = 1.5$	$\theta = 2.5$	High $\psi = 0$	Low $\psi = 2$	$\Omega = 1, \Theta = 0$	$\gamma = 0.0001$
$\rho(\tilde{c} - \tilde{c}^*, \tilde{s})$	−0.704	−0.704	−0.704	−0.704	−0.704	−0.704	−0.137
$\rho(\tilde{y} - \tilde{y}^*, \tilde{s})$	−0.704	0.704	0.704	0.704	0.704	0.704	0.137
$\rho(\tilde{c} - \tilde{c}^*, \widetilde{\text{rer}})$	−1	−1	−1	−0.994	−1	−0.995	−0.999
$\rho(\tilde{y} - \tilde{y}^*, \widetilde{\text{rer}})$	−1	1	1	0.998	0998	1	0.999
$\sigma(\tilde{s})/\sigma(\tilde{c})$	13.35	7.76	5.75	11.73	6.64	2.48	46.84
$\sigma(\tilde{s})/\sigma(\tilde{y})$	42.34	10.02	4.93	6.64	13.39	8.99	60.44
$\sigma(\widetilde{\text{rer}})/\sigma(\tilde{c})$	8.03	4.25	2.89	6.94	3.49	0.68	4.25
$\sigma(\widetilde{\text{rer}})/\sigma(\tilde{y})$	25.47	5.48	2.47	3.93	7.04	2.45	5.48

Note: Benchmark: $\omega = 2, \beta = 0.99, \theta = 1.5, \rho = 0.74, \alpha_{\text{T}} = 0.55, \alpha_{\text{H}} = 0.72, \Omega = 0.5, \Theta = 0.55$.

between consumption and the terms of trade ($\tilde{c}_t - \tilde{c}_t^* = \frac{\delta-2}{\omega\delta}\tilde{\tau}$) into (31) and (33), we have $\tilde{y}_t - \tilde{y}_t^* = \frac{2b}{\delta} + \frac{(2-\delta)^2}{\omega\delta}\tilde{\tau}_t$. Obviously, under the complete asset market, the output differential and terms of trade are always positively correlated.

3.4. Illustrative Evidence of the Effects of Sunspot Shocks

In this section, we relax the assumption of perfectly elastic labor supply (i.e., $\psi > 0$). The purpose of this section is to explore the effect of substitutability of home-produced and foreign-produced goods, labor supply elasticity, distribution costs, and transaction costs on the relationship between exchange rates and real macro variables in some illustrative examples. Table 1 reports these estimates.

The exercise is conducted by feeding two sources of uncertainty, monetary and sunspot shocks, into the model economy and simulating the model 1,000 times over 120 periods.[14] By definition, the sunspot shocks are independent of preferences, productivity, and monetary shocks. The process of the nominal exchange rate is given by $\tilde{s}_t = (\tilde{m}_t - \tilde{m}_t^*) + \tilde{\nu}_t$. It is assumed that the monetary shocks are uncorrelated across countries and each follows a random walk process. The domestic money supply follows a random walk; i.e., $\tilde{m}_{t+1} = \tilde{m}_t + \epsilon_{g,t+1}$, where $\epsilon_{g,t+1} \sim$ i.i.d. $N(0, \sigma_m^2)$ and $\sigma_m = 1.5\%$, to match the empirical level of the log difference of M1 in the United States over the period 1975Q1–2005Q4. The foreign money supply follows an independent random walk with $\sigma_{m^*} = 3\%$ to match the average data in nine OECD countries.[15] The sunspot process follows a random walk process $\tilde{\nu}_{t+1} = \tilde{\nu}_t + \zeta_{t+1}$, where $\zeta_t \sim$ i.i.d. $N(0, \sigma_\nu^2)$, and σ_ν equals 4%, based on the empirical standard deviations of money supply and the average end-of-period exchange rate to U.S. dollars in these countries for the same time period. As shown in the previous section, $\sigma_\nu = [2\ln(1+\gamma)]^{1/2}$, implying a very small transaction cost $\gamma = 0.07\%$, comparable to the empirical transaction cost of 0.05% measured in Aliber et al. (2003).[16]

Note that the model economy is highly stylized (e.g., there is no physical capital, prices are flexible, and we are assuming a special form of shock processes) and is subject only to monetary and sunspot shocks, so we do not aim to *quantitatively* match the model predictions with the observed statistical properties of international business cycles. One should interpret the numbers as *suggestive* instead of definitive. Nevertheless, the model seems to capture some empirical observations well. For example, in the data real exchange rates exhibit excessive volatility compared to economic fundamentals: the ratio between the standard deviation of real exchange rate and that of GDP is about 4.6 and the relative volatility between nominal exchange rate and GDP is 5 [according to Chari et al. (2002), for the United States vs. the European aggregate]. Our model can easily generate relative volatility of that magnitude. Nominal and real exchange rates in the data are very highly correlated (0.98). In the model the correlation is about 0.76. In addition, in the data, $\rho(\tilde{c} - \tilde{c}^*, \tilde{\text{rer}}) = -0.35$ based on data on the United States vs. the European aggregate [Chari et al. (2002)]. In contrast to most standard models where the Backus–Smith puzzle or consumption–real exchange rate anomaly is hard to solve, our model consistently generates a negative correlation between relative consumption and the real exchange rate.[17]

Other observations are as follows. First, as explained in the last section, when domestic tradables and imported goods are not very substitutable (i.e., θ is low), the output differential responds negatively to the relative price movements. The correlation between the output differential and the terms of trade is -1 when $\theta = 0.5$. The response of world demand for home goods is outweighed by a strong negative income effect, and both relative output and net exports would decrease with currency depreciation. Second, as illustrated in the previous section, when θ is low, exchange rates become much more volatile than consumption and output. Note that in this model under flexible prices, money has no real effects on the economy. Thus, sunspot shocks are the only driving force of fluctuations in consumption and output. Given that $\tilde{s}_t = (\tilde{m}_t - \tilde{m}_t^*) + \nu_t$, the relative importance of monetary shocks and sunspot shocks does not depend on parameter values of θ, Θ, and Ω. This explains why the absolute values of cross-correlation between exchange rates and real variables are the same for different parameters in Table 1.

Third, when the labor supply is perfectly elastic ($\psi = 0$), nominal wage changes in proportion to money, and hence there is no direct pass-through of the exchange rate to wages. Thus, when the exchange rate appreciates, the wage does not decrease by as much as when $\psi > 0$, so that consumption is not affected as much either. However, output reacts in a different fashion. The fall in wages generates an income effect and a substitution effect, and usually the latter dominates the former, so that labor falls. When the labor elasticity is high, output responds more than when the elasticity is low, as labor input is more responsive in this case. Therefore, the relative standard deviation of output is higher.

Fourth, as shown in the previous section, when there is no distribution cost or nontradable goods sector, the relative volatility of the exchange rate to consumption and output is significantly reduced. Put differently, the presence of nontraded goods and the distribution margin dampens the responses of relative consumption and output. Finally, when the transaction cost is lower, the correlation and relative volatility between the nominal exchange rate and macro variables are both lower because the monetary shock accounts for most of the fluctuations in macro variables. However, the relationship between the real exchange rate and macro aggregates is not changed.

4. FINAL REMARKS

Standard macroeconomic models relating the exchange rate to monetary shocks or productivity shocks are often unsuccessful in reproducing the observed high exchange rate volatility. This paper introduces a different approach to explaining the excessive exchange rate volatility and the disconnect puzzle without assuming irrationality. Instead of assuming that the macroeconomic variables and the exchange rate are endogenously driven by a set of fundamental shocks, we show that part of the exchange rate swings can be caused by nonfundamental uncertainty in a general equilibrium framework. A combination of incomplete asset markets and positive transaction costs of trading in the foreign bond market is shown to allow sunspots to exist in equilibrium exchange rates, produce high exchange rate volatility, and disconnect the exchange rate from the fundamentals. In this sense, the exchange rate breeds its own shock, as well as swings in consumption and output, and can potentially reduce welfare. This paper shows, however, that in the presence of distribution costs and nontradable goods, the real allocation effect can be quite small. Therefore, exchange rates have little effect on macroeconomic variables.

This paper does not aim to develop a complete model that can match many of the empirical statistics on exchange rate movements or the cyclical behavior of relevant macro variables. For instance, to explain persistence, one may be required to introduce price or wage stickiness, habit formation, and an adjustment cost in capital accumulation. Yet this paper shows that with a minimum set of model assumptions—incomplete asset markets and positive transaction costs associated with holding foreign-currency-denominated bonds—the exchange rate can be free from the tight link with macroeconomic fundamentals and partly driven by nonfundamental randomness.

NOTES

1. Recently, there have been some important developments in the literature. Engel and West (2005) argue that when fundamentals follow an $I(1)$ process and investors are patient enough, the exchange rate should display near-random-walk behavior. Rey and Gourinchas (2007) show that the ratio of net exports to net foreign assets helps to predict the exchange rate in and out of sample at one quarter and beyond.

2. For example, Buiter (2000) suggests that a major benefit of EMU membership for the United Kingdom might be to escape from the destabilizing effects of the foreign exchange market. In particular, he commented, "I view exchange rate flexibility as a source of shocks and instability as well as (or even rather than) a mechanism for responding effectively to fundamental shocks originating elsewhere."

3. This finding speaks to the Meese and Rogoff (1983) result that a random walk model beats fundamental models of exchange rate determination. It implies that in fact random walk exchange rate does not have to be inconstant with equilibrium monetary models.

4. Various approaches have been introduced in the literature to explain the low exchange rate pass-through (e.g., sticky prices, price-to-market, distribution service).

5. Benigno (2009) and Benigno and Thoenissen (2003) rationalize the cost on foreign bond stock by considering foreign-owned intermediaries in the foreign asset market who apply a spread over the risk-free interest rate when borrowing or lending to home agents in the foreign currency. However, the spread is assumed to depend on the net foreign asset position of the home country in their papers.

6. The fact that the nominal exchange rate is determined simply by the ratio of money supply is an artifact of the assumption that real money balances enter the utility function logarithmically. If we allow the real money balance to enter the utility function in CRRA form, such as $(M_t/P_t)^{1-\varphi}/(1-\varphi)$, the stochastic discount factor is changed to $\tilde{Q}_{t,t+1} = \beta(\frac{M_{t+1}}{M_t})^{-\varphi}(\frac{P_{t+1}}{P_t})^{\varphi-1}\frac{1-q_t}{1-q_{t+1}}$. Going through the same steps, we can define the fundamental-determined nominal exchange rate as $\tilde{S}_t^f = (\frac{M_t}{M_t^*})^{\varphi}(\frac{P_t}{P_t^*})^{1-\varphi}(\frac{1-q_t}{1-q_t^*})$. Injecting the same type of sunspots into the model, we speculate that the nominal exchange rate is $\tilde{S}_t = \tilde{S}_t^f \nu_t$, where ν_t satisfies the same martingale processes as in (19). It is easy to verify that the exchange rate constructed in this way satisfies the equilibrium conditions (16) and (17). Therefore, our previous results are not sensitive to our particular preference.

7. Note that a random walk sunspot process is not the only stochastic process that gives rise to this positive relationship between the exchange rate volatility and the size of the transaction cost. There could be other probability structures that satisfy the two martingale process—for example, $\Pr\{\nu_{t+1} = \nu_t(1+\gamma)(1+\varepsilon)\} = \Pr\{\nu_{t+1} = \nu_t(1+\gamma)(1-\varepsilon)\} = 1/2$, where $\varepsilon = \sqrt{1-(1+\gamma)^{-2}}$. The conditional standard deviation of the exchange rate is given by $\sigma_t(S_{t+1}) = S_t(g/g^*)[(1+\gamma)^2-1]^{1/2}$, which is positive as long as there exists a positive transaction cost. The volatility increases with the transaction cost and also the relative growth rate of domestic money supply.

8. Alternatively, one can include sticky prices to get fluctuations in real exchange rates from fluctuations in the relative price of traded goods, as in Chari et al. (2002).

9. That is, $\mathrm{RER}_t = \dfrac{\left[\alpha^{\mathrm{T}}([(1-\alpha_{\mathrm{H}})(1+\eta\tau_t)^{1-\theta}+\alpha_{\mathrm{H}}(\tau_t+\eta\tau_t)^{1-\theta}]^{\frac{1}{1-\theta}})^{1-\rho}+(1-\alpha^{\mathrm{T}})\tau_t{}^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[\alpha^{\mathrm{T}}([\alpha_{\mathrm{H}}(1+\eta)^{1-\theta}+(1-\alpha_{\mathrm{H}})(\tau_t+\eta)^{1-\theta}]^{\frac{1}{1-\theta}})^{1-\rho}+(1-\alpha^{\mathrm{T}})\right]^{\frac{1}{1-\rho}}}$.

10. Let $\tilde{x} = \ln(X/\bar{X})$, where $\bar{X}$ is the value associated with a steady state symmetric economy. In a steady state symmetric equilibrium, no one would optimally hold any bonds, either home or foreign bonds. Thus, equations (18) and (19) do not need to hold.

11. The decomposition of the real exchange rate shows that the productivity fluctuation across countries and across sectors within countries also drives swings in real exchange rate: $\mathrm{rer}_t = [1 + \Omega(1-\Theta)(2\alpha_{\mathrm{H}}-2))(\tilde{\nu}_t + \tilde{a}_t - \tilde{a}_t^*)] + [1-(1-\Theta)\Omega](\tilde{a}_t^{\mathrm{N}} - \tilde{a}_t^{*\mathrm{N}})$. But for the purpose of the paper, we choose to focus on the role of sunspot movements and ignore the changes in productivity differentials.

12. This analysis also sheds light on the importance of the dynamic approach. Given equation (32), in a static model $\tilde{nx}_t = 0$, this restriction from the goods market implies that sunspots cannot exist in theat equilibrium. Hence, the nominal exchange rate is only determined by monetary policies. Again, the incomplete market is necessary for this result because all the relationships derived here are based on the household budget constraint. The budget constraints in a complete market model would play no role in consumption determination, because the complete state-contingent assets would guarantee an optimal allocation for every state.

13. For the United States, the correlation is –0.33; the median correlation in the sample is –0.19 using HP-filtered data and –0.09 using first-difference data.

14. Following Backus et al. (1995), set θ equal to 1.5, which is in between the range of estimates using macro and micro evidence [see Feenstra et al. (2012)]. The elasticity of substitution between tradable and nontradable goods ρ is set to be 0.74 as estimated by Mendoza (1995) for a sample of industrialized countries. The values of the share of traded goods α_{T} and the share of domestic goods α_{H} are from Corsetti et al. (2008) and are set to 0.55 and 0.72, respectively. The share of traded goods in the consumption basket Ω is 0.5, as suggested by Stockman and Tesar (1995). Following Anderson and van Wincoop (2004), distribution margin Θ is set equal to 0.55, which implies that the distribution cost $\eta = 1.2$.

15. The countries included in the M1 data (seasonally adjusted, if available) are Australia, Austria, Canada, France, Germany, Japan, New Zealand, Spain, and Sweden. The data for the European countries end in 1998Q4. The data are from the IMF International Financial Statistics.

16. There is no direct measure of transaction costs in the foreign exchange market. Transaction costs in the FX are implicit and are collected by broker-dealers in the spreads between the buy and sell prices of foreign exchange. Moreover, they can differ across different currencies, across different transaction sizes, and between different buyers and sellers. Using daily prices per unit of four major currencies in sU.S. dollars—the British pound, Deutsche mark, Japanese yen, and Swiss franc—from 1977 to 1997, Aliber et al. (2003) estimate that the transaction costs incurred by large commercial banks on average were about 0.05% and are likely to be smaller than the ones faced by other banks.

17. The current correlation is perfectly negative because both relative consumption and the real exchange rate are driven only by sunspot shocks. Adding productivity shocks has the potential to bring the quantitative result closer to the data. But it is by keeping the shock structure simple that its interactions with the mechanism are made most transparent.

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